

آموزشگاه علمی میرابی



Mirabi Educational Center

جزوه آموزشی ریاضی SAT

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SAT Mathematics

Arithmetics

Union of two Set	اجتماع دو مجموعه
Intersection	اشتراک
Solution Set	مجموعه جواب
Absolute Value	قدر مطلق
Arithmetics	حساب

Example) If A is the solution set of the equation $x^2 - 4 = 0$ and B is the solution set of equation $x^2 - 3x + 4 = 0$, how many elements are in the union of the two sets?

Solution: $A = \{-2, 2\}$

$$B = \{1, 2\}$$

$$A \cup B = \{1, 2, -2\}$$

therefore there are 3 elements in the union.

Operation	Symbol	Result
Addition	+	Sum
Subtraction	-	difference
Multiplication	×	Product
Division	÷	quotient

Example) What's the product of all the integers from -3 to 6 ?

Solution: 0

Number Sets

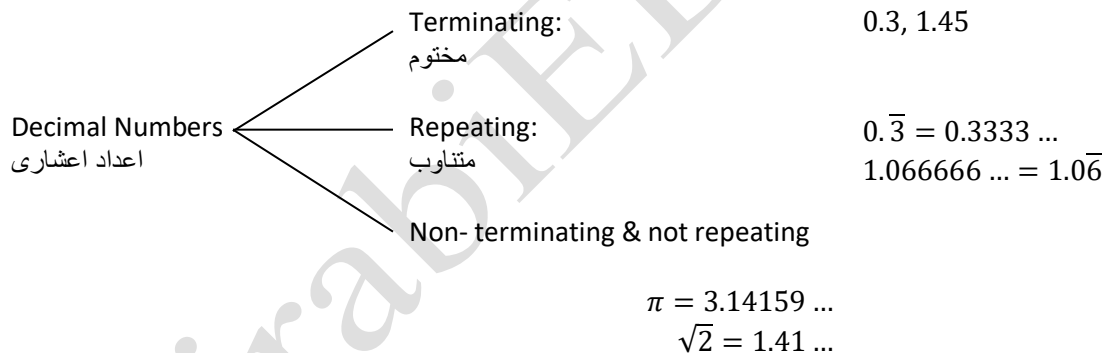
Integers	اعداد صحيح	\mathbb{Z}
Positive Integers	اعداد طبيعي	\mathbb{N}
Rational Numbers	اعداد گویا	\mathbb{Q}
Irrational Numbers	اعداد گنگ (اصم)	\mathbb{Q}'
Real Numbers	اعداد حقيقي	\mathbb{R}

Example) which of the following is not a rational number?

$\sqrt{4}$ 0.3 0.66666 ... π 17.614

Solution: π is irrational and all other choices are rational.

Every number that could be written in a fraction is form is a rational number.



from three set of decimal numbers above, the first 2 are rational numbers and the 3rd ones are irrational.

Example) why repeating decimals like $0.\bar{3}$ & $1.0\bar{6}666 \dots$ are rational?

$$x = 0.\bar{3} = 0.3333 \dots$$

$$10x = 3.3333 \dots$$

subtraction: $9x = 3 \Rightarrow x = \frac{3}{9} = \frac{1}{3}$

$$x = 1.066666 \dots$$

$$\begin{cases} 10x = 10.66666 \dots \\ 100x = 106.6666 \end{cases}$$

Subtract: $90x = 96 \Rightarrow x = \frac{96}{90}$

Reciprocal	معكوس يك عدد	Prime number	عدد اول
Opposite	قرينه	Composite number	عدد مركب
Even	زوج	Divisor	مقسوم عليه
Odd	فرد	Factor	عامل
Consecutive integers	اعداد صحيح متوالی	Multiple	مضرب

Prime Number: Positive Integers that have exactly two divisors.

$$2, 3, 5, 7, 11, 13, \dots$$

Composite Number: Positive Integers that have more than two divisors.

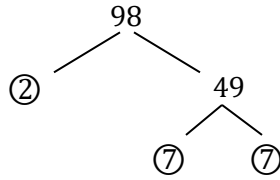
$$4, 6, 8, 9, 10, 12, \dots$$

* 1 is neither prime nor composite.

Prime Factorization: every integers greater than 1 that is not a prime, Can be written as a product of primes.

$$54 = 2 \times 3^3$$

$$240 = 2^4 \times 3 \times 5$$



Example) for any positive integer a , Let $[a]$ denote the smallest prime factor of a . which of the following is equal to $[35]$?

[10] [15] [45] [55] [75]

Solution: $[35] = 5$ $[10] = 2$ $[15] = 3$ $[55] = 5$

Correct answer is [55]

If $N = P_1^{\alpha_1} \cdot P_2^{\alpha_2} \cdot P_3^{\alpha_3} \dots P_k^{\alpha_k}$

Number of divisors of $N = (\alpha_1 + 1)(\alpha_2 + 1) \dots (\alpha_k + 1)$

Example) a) How many divisors does 12'000 have?

b) How many of them are prime?

c) How many of them are composite?

Solution: $12000 = 2^2 \times 3 \times 2^3 \times 5^3 = 2^5 \times 3 \times 5^3$

a) No of divisors = $(5 + 1)(1 + 1)(3 + 1) = 48$

b) 3 because, in prime factorization, there are 3 prime numbers.

c) $48 - 3 - 1 = 44$

Because 1 is neither prime nor composite, we subtracted one more.

Example) How many zeros at the right, does 17! Have?

Solution: 3, because of factors of 5. In fact zeros at the right are produced by factors of 2 & 5. But multiples of 5 are the bottle neck; Which are 5, 10, 15

Example) how many digits does $4^8 \times 3^3 \times 5^{15}$ have?

$$2^{16} \times 3^3 \times 5^{15} = 2 \times 3^3 \times 10^{15}$$

It has 17 digits, because it'll be a 54 and 15 zeros ahead of it.

Least Common Multiple (LCM)=
کوچکترین مضرب مشترک

To find LCM of two or more numbers by their prime factorization, we multiply all the bases, when bases are common we take the greatest power.

Greatest Common Divisor (GCD)=
(factor)(GCF)
بزرگترین مقسوم علیه مشترک

To find gcd of two or more numbers by their prime factorization, we multiply the common bases with lowest power.

Example) find LCM & GCD of 60 and 378.

Solution:

$$\begin{array}{l} 60 = 2^2 \times 3 \times 5 \\ 378 = 2 \times 3^3 \times 7 \end{array} \left. \vphantom{\begin{array}{l} 60 \\ 378 \end{array}} \right\} \Rightarrow \begin{array}{l} LCM = 2^2 \times 3^3 \times 5 \times 7 \\ GCD = 2 \times 3 \end{array}$$

Example) We have 8 Liters of milk , 12 Liters of Mineral water and 20 liters of juice. We want to pour them into bottle of the same size. What is the minimum number of bottles needed?

Solution: If we use 1 L bottles we'll need

8 bottles for milk
12 " " water
20 " " juice

If we use 2 L bottles, the number of bottles will decrease.